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An analysis method for the vibration signal with amplitude modulation in a bearing system

Yuh-Tay Sheen*

Department of Mechanical Engineering, Southern Taiwan University of Technology, 1 Nan-Tai Street, Yung Kang City, 710 Tainan County, Taiwan, ROC

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Abstract

In this paper, an envelope detection method for the vibration signal with amplitude modulation is proposed. The method is to filter the vibration signal through a designated band-pass filter. According to the resonance of the vibration signal in the passband, the filtered signal would be decomposed into a sinusoidal function basis at the resonant frequency. Under the assumption of a stepwise function for the envelope signal, the envelope signal could be derived by the linear least-squares analysis. In addition, the filtered signal could be easily reconstructed from the envelope signal. Furthermore, the logarithmic transformation is applied in this envelope signal, the defect frequencies due to bearing defects could be more enhanced by suppressing the corresponding sidebands. However, the logarithmic transformation would have little effect in the spectrum for a normal bearing. Accordingly, logarithmic transformation would be helpful to apply the signal processing method in enhancing the defect frequencies for the bearing defect diagnosis.

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1. Introduction

The analysis of a vibration signal with amplitude modulation is usually based on the high-frequency resonance technique [1]. The phenomenon of amplitude modulation is because a high-frequency carrier signal is varied by a low-frequency modulating signal. Thus, the modulated signal could be the product of the modulating signal and the carrier signal. The modulating signal is the impact caused by defects of a bearing and could be represented by bursts of exponentially decaying vibration. Thus, its spectrum would be expanded in a frequency band. The carrier signal is a combination of the resonance frequencies of the bearing or even of the mechanical system, whose frequency is generally higher than 2 kHz. The frequency of modulating signal would also be expanded in frequency band whose center frequency would be at the frequencies of carrier signal. To deal with the amplitude modulation, the high-frequency resonance technique is tried to derive the low-frequency modulating signal from the modulated signal by means of filtering out the high-frequency carrier signal. In the range of a high-frequency system resonance, this technique takes advantage of the absence of low-frequency

E-mail address: syt@mail.stut.edu.tw.

^{*}Tel.: +88662533131x3522; fax: +88662425092.

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mechanical noise to demodulate a vibration signal and, therefore, provides a low-frequency demodulated signal with a high signal-to-noise ratio [2].

The procedure of high-frequency resonance technique could be implemented in two steps. First, a band-pass filter is applied to the vibration signal around a selected high-frequency band with the center at a chosen resonant frequency of mechanical system. Second, the band-passed signal is demodulated with an envelope detection process. In the past, the envelope detection for band-pass filtered signal could be practically implemented by using a rectifier and a low-pass filter. The low-pass filter is used to cancel the high-frequency components and retain the defect information. However, the derived envelope signal would not guarantee all to be positive. But it is impossible to be negative for an actual envelope signal. It is a deficiency that arises because of application of the low-pass filter in the envelope detection. In addition, the low-pass filter would distort the envelope signal in both amplitude and phase. On the other hand, the Hilbert transform is a more precise method to apply in the technique for the envelope detection. There are three types of transformation: the filter-based Hilbert transform, the FFT-based Hilbert transform and the wavelet-based Hilbert [3]. It is shown that the phase distortion could occur in the filter-based Hilbert transform [3] and the leakage error would exist in the FFT-based Hilbert transform [4]. The wavelet-based Hilbert transform has better properties and reduces both of the above two problems. But, more computing time is required in the envelope detection.

On the other hand, it is shown [5] that the spectrum for the envelope signal has a pattern of sidebands around the defect frequency and its harmonics. The sidebands are related to the frequencies of loading distribution [5] and transmission path [6]. However, the envelope spectrum for a normal bearing would be possible to show a similar defect pattern of the main peaks with equal frequency spacing sidebands under large preload [7] or at high running speed [8], except the main peaks in the spectrum for a normal bearing are lower [9]. In addition, for a defect bearing the magnitude of sidebands could be enhanced due to a non-uniform loading [9]. Thus, the difficulty of bearing defect diagnosis would be raised under non-uniform loading condition.

In this paper, an envelope detection method for the vibration signal with amplitude modulation is proposed to achieve a non-negative envelope. The signal processing method is according to the resonance of vibration signal, a band-pass filter would be designated to filter the vibration signal. Then, the filtered signal would be decomposed into a sinusoidal function basis at the resonant frequency, which could be derived from the highest peak in the spectrum for the filtered signal. For the envelope detection of the vibration signal, a calculating algorithm is presented. Based on the envelope signal, the algorithm could be reversed to reconstruct the vibration signal with noise rejection.

For the purpose of enhancing the defect frequency, the logarithmic transformation is applied in the envelope signal and would be enhancing the defect frequency more for a defect bearing. On the other hand, the logarithmic transformation would have little effect on the envelope signal of a normal bearing. Consequently, this method would be effective in enhancing the amplitude difference of spectra between a defect bearing and a normal one.

2. Signal processing for vibration signal with amplitude modulation

In the diagnosis of a mechanical system, the vibration of a defect bearing is typical of amplitude modulation. Because of the amplitude modulation occurring in the measured vibration signals, the frequency-translation property would be presented in the vibration spectra. This phenomenon of amplitude modulation arises because a high-frequency carrier signal is varied by a low-frequency modulating signal. Thus, the modulated signal could be the product of the modulating signal with the carrier signal. Moreover, the modulating signal is the impact caused by bearing defects and could be represented by bursts of exponentially decaying vibration. Therefore, its spectrum would be expanded in a frequency band and difficult to find the characteristic frequency of modulating signal.

2.1. Amplitude modulation of bearing vibration

In a bearing system, the carrier signal could be a combination of the resonant frequencies of the bearing or even of the mechanical system, and thus the vibration signal with amplitude modulation could be represented as [5-7]

$$v(t) = \sum_{l=1}^{L} \left(\sum_{m=1}^{md} \int_{-\infty}^{t} d_{m}(\tau) q_{m}(\tau) a_{lm}(\tau) e^{-\sigma_{l}(t-\tau)} \cos(2\pi f_{l}(t-\tau) + \theta_{lm}) d\tau + \sum_{n=1}^{nr} \int_{-\infty}^{t} g_{n}(\tau) q_{n}(\tau) a_{ln}(\tau) e^{-\sigma_{l}(t-\tau)} \cos(2\pi f_{l}(t-\tau) + \theta_{ln}) d\tau \right),$$
(1)

where the two summation terms describe the vibrations of defect components and normal components, respectively. In the first term of Eq. (1), md is the defect number, $d_m(t)$ depicts the defect impulse train of contact and is the modulating signal, $q_m(t)$ describes the dimension information of defect and the sensitivity of striking energy, and $a_{lm}(t)$ is a characteristic function of transmission path. In the other term, nr is the roller number, $g_n(t)$ describes the surface functions of normal bearing components with respect to roller n and could also be a modulating signal, $q_n(t)$ is the equivalent stiffness of roller n and is a function of the structure stiffness and the oil film stiffness, and $a_{ln}(t)$ is the transmission path function which describes the vibration strength excited by roller n. It should be noted that the surface irregularity could be enhanced under large preload or at high running speed and the surface function $g_n(t)$ would be similar to an impulse train, otherwise the surface function $g_n(t)$ would be random. For the other variables, σ_l and f_l are respectively the exponential frequency and the carrier frequency, and would be the intrinsic characteristics of the system. L is the quantity of vibration mode of the system. θ_{lm} and θ_{ln} are the initial angles for the amplitude modulation. It should be noted that the frequencies of modulating signal $d_m(t)$ and $g_n(t)$ would be always much less than that of the carrier signal, and the frequencies of modulating signal $d_m(t)$ and $g_n(t)$ are higher than those of $q_m(t)$, $a_{lm}(t)$, $q_n(t)$ and $a_{ln}(t)$. Accordingly, the modulated signal would be expanded in frequency band whose center frequency would be at the frequencies of carrier signal. This phenomenon is named the amplitude modulation. Based on such an amplitude modulation model, the high-frequency resonance technique is proposed to derive the low-frequency modulating signal from the modulated signal by means of filtering out the high-frequency carrier signal [2,5,6].

Suppose that the impulse responses due to impacts $d_m(t)$ completely die out in a time interval between two consecutive contacts, and the resonant frequency f_l is high. The impact energy due to the surface irregularity $g_n(t)q_n(t)$ would be much less than that due to the bearing defects. The vibration signal of defect bearing could be operated through a band-pass filter to derive one vibration mode and then the enveloping transformation is applied to retrieve a demodulated signal. Such a process is named as the high-frequency resonance technique. Accordingly, the envelope signal of the *l*th mode vibration could be written as

$$e_{l}(t) = \sum_{m=1}^{md} u_{m}(t)q_{m}(t)a_{lm}(t) + \sum_{n=1}^{nr} w_{g_{n},q_{n},a_{ln}}(t)$$

with $u_{m}(t) = e^{-\sigma_{l}t'}, \quad t' = mod(t, 1/f_{dm}),$ (2)

where mod $(t, 1/f_{dm})$ represents a residue of t, and f_{dm} is the frequency of impulse train $d_m(t)$. For a defect bearing, the first term in Eq. (2) would dominate the vibration energy. The spectrum for the envelope signal would show a pattern of the modulating signal frequency and its harmonics with equal frequency spacing sidebands which are induced by the frequencies of $q_m(t)$ and $a_{lm}(t)$. For a normal bearing, the vibration signal in the first terms of Eqs. (1) and (2) would be eliminated. In addition, the surface irregularity of bearing component could be enhanced under large preload [6] or at high running speed [7] and the surface function $g_n(t)$ would be similar to an impulse train, otherwise the surface function $g_n(t)$ and (t) under large preload or at high running speed, the periodic characteristics in the second term of Eq. (2) could be the same as that in the first term. Therefore, the spectrum for a normal bearing would be possibly similar to that for a defect bearing,



Fig. 1. An impulse response decomposes into a sinusoidal function at its resonant frequency.

except the main peaks, i.e. the defect frequency and its harmonics, in the spectrum for a normal bearing are lower [8]. If the running condition of a normal bearing is not the case, the spectrum would be a random pattern.

2.2. Least-squares analysis for envelope detection

In the following, the linear least-squares analysis method is proposed to estimate the envelope of a vibration signal. According to Eq. (1), the *l*th mode of the vibration signal could be filtered out by a designated bandpass filter and would be represented as

$$v_{l}(t) = \sum_{m=1}^{md} u_{m}(t)q_{m}(t)a_{lm}(t)\cos\left(2\pi f_{l}t + \theta_{lm}\right) + \sum_{n=1}^{nr} w_{g_{n},q_{n},a_{ln}}(t)\cos\left(2\pi f_{l}t + \theta_{ln}\right)$$

= $c_{1}(t)\cos(2\pi f_{l}t) + c_{2}(t)\sin(2\pi f_{l}t),$ (3)

where $c_1(t)$ and $c_2(t)$ are the coefficients for the vibration signal $v_l(t)$ mapping to a sinusoidal function basis at its resonant frequency f_l . It is noted that the envelope signal $e_l(t)$ would become smoother, and the envelope signal could thus be approximated by a stepwise function. Moreover, it is assumed that every time period of the carrier signal is divided into two parts for the envelope detection. Accordingly, the above signal could be expressed in a discrete mode:

$$v_l\left(\frac{i}{f_s}\right) = c_1\left(\frac{j}{2f_l}\right)\cos\left(2\pi f_l\frac{i}{f_s}\right) + c_2\left(\frac{j}{2f_l}\right)\sin\left(2\pi f_l\frac{i}{f_s}\right),\tag{4}$$

where *i* and f_s is the sampling point and the sampling frequency of the vibration signal, respectively. *j* is the sampling point of the envelope signal. It should be noted that the resonant frequency f_l could be derived from the highest peak in the spectrum of the band-pass filtered signal $v_l(t)$. In addition, the sampling frequency f_s must be at least four times higher than the resonant frequency f_l . As shown in Fig. 1, there are more than two equations to solve the unknown coefficients $c_1(j/2f_l)$ and $c_2(j/2f_l)$. If the number of data points is *h* over the *k*th

half-period of the vibration signal $v_l(t)$, these points could be expressed in the matrix form:

$$\begin{bmatrix} \cos\left(2\pi f_l \frac{h(k-1)+1}{f_s}\right) & \sin\left(2\pi f_l \frac{h(k-1)+1}{f_s}\right) \\ \cos\left(2\pi f_l \frac{h(k-1)+2}{f_s}\right) & \sin\left(2\pi f_l \frac{h(k-1)+2}{f_s}\right) \\ \cos\left(2\pi f_l \frac{h(k-1)+3}{f_s}\right) & \sin\left(2\pi f_l \frac{h(k-1)+3}{f_s}\right) \\ \vdots & \vdots \\ \cos\left(2\pi f_l \frac{hk}{f_s}\right) & \sin\left(2\pi f_l \frac{h(k-1)+3}{f_s}\right) \end{bmatrix} \begin{bmatrix} c_1\left(\frac{k}{2f_l}\right) \\ c_2\left(\frac{k}{2f_l}\right) \end{bmatrix} = \begin{bmatrix} v_l\left(\frac{h(k-1)+1}{f_s}\right) \\ v_l\left(\frac{h(k-1)+3}{f_s}\right) \\ v_l\left(\frac{h(k-1)+3}{f_s}\right) \\ \vdots \\ v_l\left(\frac{h(k-1)+3}{f_s}\right) \end{bmatrix}.$$
(5)

By a simplified expression, the above equation would be rewritten as

$$[M_k]_{h\times 2} \begin{bmatrix} c_1\left(\frac{k}{2f_l}\right) \\ c_2\left(\frac{k}{2f_l}\right) \end{bmatrix} = [V_{lk}]_{h\times 1}.$$
(6)

The solution of the above equation in two unknowns could be obtained in a linear least-squares sense by simple equations

$$\begin{cases} \begin{bmatrix} c_1\left(\frac{k}{2f_l}\right)\\ c_2\left(\frac{k}{2f_l}\right) \end{bmatrix} = [M_k]_{h\times 2}^{-1}[V_{lk}]_{h\times 1}, & \text{for } h = 2, \\ \begin{bmatrix} c_1\left(\frac{k}{2f_l}\right)\\ c_2\left(\frac{k}{2f_l}\right) \end{bmatrix} = \left([M_k]_{h\times 2}^{\mathrm{T}}[M_k]_{h\times 2}\right)^{-1}[M_k]_{h\times 2}^{\mathrm{T}}[V_{lk}]_{h\times 1}, & \text{for } h > 2, \end{cases}$$

$$\tag{7}$$

where T denote the transport of a matrix. Accordingly, the envelope signal in Eq. (2) could be derived from the following equation:

$$e_l\left(\frac{j}{2f_l}\right) = \sqrt{c_1\left(\frac{j}{2f_l}\right)^2 + c_2\left(\frac{j}{2f_l}\right)^2}.$$
(8)

Based on the envelope signal, the vibration signal with noise rejection could also be reconstructed by Eq. (4).

In comparison with the demodulated signal derived from Hilbert transform, the above envelope detection method brings two advantages. One is that it could derive a positive envelope signal. The other is that it is very easy to calculate. The envelope detection method is directly to decompose the vibration signal according to its resonant frequency to derive the corresponding envelope signal, while the wavelet decomposition and reconstruction could not accurately determine the carrier frequency because the wavelet is a sub-band function with a range of frequencies. Thus, the above analytical expression is very difficult to be accomplished by the wavelet transform. In addition, the vibration signal could be easily reconstructed from the envelope signal. However, the reconstruction of vibration signal by using wavelet transform would be more difficult.

2.3. Spectral characteristics with logarithmic transformation

Because the spectrum for the envelope signal $e_i(t)$ for a normal bearing would be possible to show a similar defect pattern of the main peaks with equal frequency spacing sidebands except the main peaks in the

spectrum for a normal bearing are lower [8]. For a defect bearing, the amplitude of sidebands could be enhanced due to a non-uniform loading. Thus, the difficulty of bearing defect diagnosis would be raised for a non-uniform loading condition. In order to reduce the effect on the sidebands of the envelope spectrum, the logarithmic transformation is proposed in the following, and it would be applied to the signal processing of the envelope signal $e_l(t)$ of Eq. (2).

If the impulse responses due to impacts $d_m(t)$ are completely die out in a time interval between two consecutive contacts, the second term of Eq. (2) could be regarded as a noise $w_{lm}(t)$ in the time interval. Thus, Eq. (2) is rewritten as

$$e_{l}(t) = \sum_{m=1}^{md} (u_{m}(t)q_{m}(t)a_{lm}(t) + w_{lm}(t)),$$

where $\sum_{m=1}^{md} w_{lm}(t)$ is the vibration period of $\sum_{n=1}^{nr} w_{g_n,q_n,a_{ln}}(t)$ between the two consecutive contacts. The logarithmic transformation on both sides of the above equation would be expressed as

$$\log(e_l(t)) = \sum_{m=1}^{md} \left(\log(u_m(t)) + \log(q_m(t)) + \log(a_{lm}(t)) + \log\left(1 + \frac{w_{lm}(t)}{u_m(t)q_m(t)a_{lm}(t)}\right) \right).$$
(9)

The periodic characteristics of $log(u_m(t))$ would correspond to the defect frequencies. The periodic characteristics of loading and transmission path would show peaks at the frequencies of their corresponding harmonics. Thus, it would be possible to diminish the sidebands in the spectrum, if $w_{lm}(t)$ is very small in comparison with $u_m(t) q_m(t) a_{lm}(t)$. For each time interval between two consecutive contacts, the impact energy due to the surface irregularity $g_n(t)q_n(t)$ would be generally less than that due to the bearing defects. The 4th term in Eq. (9) could be regarded as a noise term, and would be bounded in the small range

$$0 < \log\left(1 + \frac{w_{lm}(t)}{u_m(t)q_m(t)a_{lm}(t)}\right) < 0.693.$$
⁽¹⁰⁾

Thus, the vibration energy in the spectrum due to the 4th term in Eq. (9) would be small. On the other hand, the magnitude of the first three terms in Eq. (9) are bounded in large ranges $[-\infty, \log(\max(u_m(t)))]$, $[-\infty, \log(\max(q_m(t)))]$, and $[-\infty, \log(\max(a_{lm}(t)))]$, respectively, where max() is a function to take the maximal value. Hence, the vibration energy in the spectrum due to the first three terms in Eq. (9) would be possibly large. Accordingly, higher the energy of defect impulses or lower the energy due to the surface irregularity would enhance the effect of reducing the sidebands in the spectrum around the main peaks by the logarithmic transformation. However, it should be noted that under multiple defect condition the impulse responses due to defect impacts could be not complete die out between two consecutive contacts. Accordingly, the effect of reducing the sidebands around the main peaks by the logarithmic transformation could be diminished under multiple defect condition.

3. Experimental study

In the following, the applications of the proposed method on the vibration signals of tapered roller bearings (SKF type 32208) are studied. The electrical-discharge machining method is applied to produce artificial defects on the surface of bearing components which are roller, outer race and inner race. The defect sizes are described in Table 1 and the description of the test rig is shown in Fig. 2. The vibration signals are measured on the housing of the test bearing by mounting an accelerometer with sensitivity 105.5 mV/g. The measured

Table 1 Defect sizes of defective bearings

fect type Defect size (length × width × depth)	
Roller defect	16 mm × 0.15 mm × 0.1 mm
Outer-race defect	14 mm × 0.15 mm × 0.1 mm
Inner-race defect	18.5 mm × 0.15 mm × 0.1 mm



Fig. 2. Schematic of the test rig.



Fig. 3. Spectra for a roller defect bearing running at 2400 rev/min: (a) power spectrum, (b) power spectrum for the band-pass filtered signal and (c) spectrum for the signal reconstructed from the envelope signal.

direction is radial to the shaft. The tested bearings run at 800 and 2400 rev/min. The sampling rate of vibration signal is 25 kHz. According to the spectra of bearing vibrations, the frequency band from 3 to 5 kHz is chosen. For the purpose of demonstration, the spectra of an envelope signal with (without) the logarithmic transformation is named logarithmic spectrum (envelope spectrum). To estimate the spectra, 8192 points of data are calculated and 20 estimations are averaged.

In Fig. 3, the spectra for a roller defect bearing running at 2400 rev/min are the power spectrum for the measured signal, the spectrum for the measured signal passing through the band-pass filter and the spectrum for the signal reconstructed from the envelope signal in Eq. (8). According to Fig. 3(b), the highest peak would be the resonant frequency in the filtering pass-band. In comparison Fig. 3(c) with Fig. 3(b), they are almost the same. It would prove that the envelope signal in Eq. (8) could be applied in reconstruction of vibration very well. Fig. 4 shows the filtered signal for Fig. 3, the envelope signal of Fig. 4(a), and the reconstructed signal from Fig. 4(b). From the above, it is shown that the proposed envelope detection method could be effectively applied in the envelope analysis of the vibration signal with amplitude modulation.

According to the dimension of tested bearings, the characteristic frequencies for roller defect, outer- and inner-race defect are shown in Table 2. The envelope and the logarithmic spectra for a roller defect bearing running at 800 rev/min are shown in Fig. 5. In comparison Fig. 5(b) with Fig. 5(a), the sidebands around the main peaks are much reduced, especially those around the harmonics. Fig. 6 is the spectra for a roller defect bearing the sidebands in Fig. 5 is more obvious than that in Fig. 6. The reason might be that the surface irregularity of the bearing is enhanced at high running speed to diminish the effect of reducing the sidebands. Figs. 7 and 8 show the spectra for an outer-race defect bearing running at 800 and 2400 rev/min, respectively. The sidebands in the envelope spectra for both figures are very little. Thus, the sidebands in the logarithmic spectra for both figures are almost not visible.



Fig. 4. (a) Band-pass filtered signal for Fig. 2(a) and (b) envelope signal of (a), and (c) reconstructed signal from (b).

Bearing type	Running speed		
	800 rev/min	2400 rev/min	
Roller defect bearing (Hz)	77.4	232.3	
Outer-race defect bearing (Hz)	94.9	284.7	
Inner-race defect bearing (Hz)	131.8	395.3	





Fig. 5. Spectra for a roller defect bearing at running speed 800 rev/min: (a) envelope spectrum and (b) logarithmic spectrum.

Figs. 9 and 10 show the spectra for a multiple defect bearing running at 800 and 2400 rev/min, respectively. Comparing Fig. 9(b) with Fig. 9(a), sidebands around main peaks due to both the outer-racer defect and the inner-race defect are not significantly reduced. The reason could be that the impulse responses due to defect impacts could not completely die out before the next contact. Furthermore, it should be noted that in Fig. 9 the sidebands around the main peaks due to roller defect seem not to be reduced at all. It could be because the vibration energy of the roller defect impacts in comparison with that induced by the surface irregularity is small enough to eliminate the effect of reducing the sidebands. In Fig. 10, there is a similar result as that in Fig. 9. Fig. 11 shows the measured signal and its corresponding envelope signal for Fig. 10. As shown in Fig. 11(b), it could be obviously found that there are many impulse responses are overlapping. For the purpose of contrast, the spectra for a normal bearing running at 800 and 2400 rev/min are shown in Figs. 12 and 13, respectively. In both figures, the logarithmic spectra are similar to the envelope spectra. It would prove that the logarithmic transformation has little effect on the envelope signal of the normal bearing.

From the above results, it is shown that the logarithmic transformation could effectively suppress the sidebands in the spectrum for a defect bearing. But it has little effect in the spectrum for a normal bearing. Thus, the main peaks due to bearing defects could be enhanced, and it would be helpful to apply the signal



Fig. 6. Spectra for a roller defect bearing at running speed 2400 rev/min: (a) envelope spectrum and (b) logarithmic spectrum.



Fig. 7. Spectra for an outer-race defect bearing at running speed 800 rev/min: (a) envelope spectrum and (b) logarithmic spectrum.



Fig. 8. Spectra for an outer-race defect bearing at running speed 2400 rev/min: (a) envelope spectrum and (b) logarithmic spectrum.



Fig. 9. Spectra for a multiple defect bearing at running speed 800 rev/min: (a) envelope spectrum and (b) logarithmic spectrum.



Fig. 10. Spectra for a multiple defect bearing at running speed 2400 rev/min. (a) envelope spectrum and (b) logarithmic spectrum.



Fig. 11. (a) Band-pass filtered signal for Fig. 9 and (b) envelope signal of (a).



Fig. 12. Spectra for a normal bearing at running speed 800 rev/min: (a) envelope spectrum and (b) logarithmic spectrum.

processing method in the bearing defect diagnosis under the condition of non-uniform loading. However, it should be noted that the logarithmic transformation is a nonlinear transform. It is found that the amplitude in Fig. 7(b) for the defect bearing running at 2400 rev/min is less than that in Fig. 6(b) for the defect bearing running at 800 rev/min. Comparing Fig. 9(b) with Fig. 8(b), a similar condition is seen to hold. Thus, the amplitude in the logarithmic spectrum would no more represent the vibration energy.

4. Discussion

In fact, the proposed envelope detection method is another implementation method of the high-frequency resonance technique. In the past, the envelope detection for band-pass filtered signal could be implemented using a rectifier and a low-pass filter. However, the derived envelope signal would not guarantee all to be positive. The low-pass filter would distort the envelope signal in both amplitude and phase. On the other hand, the proposed method is an easy estimation method and possesses a precise estimation in both amplitude and phase. In addition, it could derive a positive envelope signal. Thus, the envelope signal could be operated through the logarithmic transformation with no difficulty.

In the proposed method, the resonant frequency must be correctly selected and could be easily derived from the highest peak in the spectrum of the band-pass-filtered signal. However, this method has a limitation that the sampling rate of the vibration signal must be at least 4 times higher than the resonant frequency. Fortunately, the performance of personal computer is very much improved now. The sampling rate could be more than 100 KHz. It would, in general, satisfy the requirement of vibration signal analysis in a mechanical system. In addition, the proposed envelope detection method could be reversed for the signal reconstruction. Based on the envelope signal, the vibration signal with noise rejection could be easily reconstructed.

No matter what the running condition is, the logarithmic transformation could effectively suppress the sidebands in the spectrum for a defect bearing, but has little effect in the spectrum for a normal bearing. Thus,



Fig. 13. Spectra for a normal bearing at running speed 2400 rev/min: (a) envelope spectrum and (b) logarithmic spectrum.

it would be helpful to apply this signal processing method in the bearing defect diagnosis. However, the logarithmic transformation is a nonlinear transform. Thus, the amplitude in the logarithmic spectrum would no more represent the vibration energy for a vibration signal.

5. Conclusion

In this paper, the envelope detection method for the vibration signal with amplitude modulation is proposed. The linear least-squares analysis algorithm is applied in the vibration signal for the envelope detection. In addition, such a signal processing method could be reversed for the signal reconstruction.

In order to enhance the defect frequency in the spectrum for a defect bearing, the logarithmic transformation is proposed to suppress its sidebands. But it has little effect in the spectrum for a normal bearing. However, it should be noted that the amplitude in the logarithmic spectrum would no more represent the vibration energy. Nevertheless, it would be helpful to apply the signal processing method in enhancing the defect frequencies for the bearing defect diagnosis.

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